Fluid Mechanics EBS 189a. Winter quarter, 4 units, CRN 52984.
Lecture TWRF 12:10-1:00, Chemistry 166; Office hours TH 2-3, WF 4-5; 221 Veihmeyer Hall.

Course Description: Axioms of fluid mechanics, fluid statics, kinematics, velocity fields for one-dimensional incompressible flow including boundary layers, turbulent flow time averaging, potential flow, dimensional analysis, and macroscopic balances to solve a range of practical problems.

Concepts: Continuum approach to deforming physical/biological systems, transport theorem integral analysis, stress vector and stress tensor analysis, microscopic and macroscopic analysis of mass as well as linear and angular momentum, downscaling for information retrieval, application of theory to solve practical problems.

Goal: To apply knowledge of mathematics, science and engineering to natural and engineered systems. To use engineering methods to identify, formulate and solve problems. Prepare for study of heat and mass transfer in physical/biological systems.

Prerequisites: PHY 9B and MTH 21D (MTH 22A and 22B recommended).

Instructor(s): Wes Wallender, Professor, 221 Veihmeyer Hall, wwwallender@ucdavis.edu, 752.0688, http: enthusiasm.ucdavis.edu (expanded course outline). Purnendu Singh, Reader.

Text: *Introduction to Fluid Mechanics*. S. Whitaker. R.E. Kreiger Publishing Co. 1982.

Grading: Two midterms 20% each, final exam 40%, homework 20% (due class period after assigned, no credit if late).

Brief Course Outline:

Axioms of Fluid Mechanics

Mass and Momentum Principles, Vector Invariance, Stress Vector, Stress Tensor.

Statics

Fluids at Rest, Forces on Submerged Surfaces.

Kinematics

Transport Theorems and Mass Conservation, Application of Macroscopic Mass Balance, Stream Function, Cauchy’s First and Second Equations, Viscous Stress, Rate of Strain and Vorticity Tensors, Physical Interpretation of the Rate of Strain and the Vorticity Tensors, Velocity Potential and Stream Function, Newton’s Law of Viscosity, the Equations of Motion, Navier Stokes Equation, Applications.

Empiricism

Dimensional Analysis, Transition and Turbulent Flow, Time Averaged Continuity and Navier Stokes Equation, Physical Interpretation of Turbulence, Eddy Viscosity and Prandtl’s Mixing Length Theory, Application to Turbulent Pipe Flow.

Macroscopic Balances and Downscaling

Bernoulli’s Equation, Moving Control Volumes and Inertial Frames, Mechanical Energy Balance, Applications, Turbulent Flow in Pipes, Friction Factors, Pipeline Design.

Prepared by Wes Wallender

Expanded Course Outline, Winter 2004.

Date	Lecture/Topic	Study	Homework
01.07	**1. Mass and momentum principles.** A continuous material body and the Euler Cut are introduced and the stress vector is shown to be a direction dependent vector. Mass, linear momentum and angular momentum conservation principles are reviewed. Relations between Newton's laws to Euler's laws are developed.	Sec. 1.3 and pages 32-36 and read Ch. 1. Review Sec. 4.3 and Ch. 13 in *Calculus and Analytic Geometry* by Stein and Barcellos, 1992.	Apply Euler's Second Law to a two particle system using Euler cuts around particle I, particle II and particles I and II. Note that for $\hat{r} \cong \hat{r}_{cm}$ $\frac{d}{dt}\int_{V_{m(t)}} \hat{r} x \rho \hat{v} dV = \frac{d}{dt}(\hat{r}_{cm} x \int_{V_{m(t)}} \rho \hat{v} dV)$ Use, $\frac{d\hat{r}}{dt} = \hat{v}$ $\frac{d}{dt}(\hat{a} x \hat{b}) = \frac{d\hat{a}}{dt} x \hat{b} + \hat{a} x \frac{d\hat{b}}{dt}$ Show that Euler's Second Law is restricted to the strong form of Newton's Third Law.
01.08	**2. Vector invariance.** Reference and inertial frames are defined. Transformations of base vectors and components of vectors arise from invariance. The summation and free index notation as well as the Kronecker delta are powerful tools used in mechanics.	Sec. 1.6	1. Using vector invariance, show how primed basis vectors can be transformed into unprimed basis vectors. 2. Show how to find the transformation of vector components from the primed to the unprimed coordinate system. (Hint: use orthogonality condition shown in class). 3. Using vector invariance, show how unprimed basis vectors can be transformed into primed basis vectors. Find another orthogonality condition starting with the primed basis vectors.
01.09	**3. Stress vector**. Stress is a doubly directed quantity and Cauchy's lemma reveals its nature and supports the development of Cauchy's Fundamental Theorem, developed in the next lecture. The projected area theorem as well as the projection operator (tensor), with which you are already familiar, are tools necessary to understand the development of Cauchy's Fundamental Theorem.	Sec. 4.2 and pages 118-119. Stein and Barcellos ch. 12.	Show that $\mathbf{a} \bullet \mathbf{B} = \mathbf{B}^T \bullet \mathbf{a}$, in which **a** is a first order tensor (a vector) and **B** is a second order tensor, using mixed notation.
01.13	**4. Stress tensor**. Cauchy's Fundamental Theorem provides the relationship between the stress tensor and the stress vector.	Sec. 4.2	

01.14	**5. Static Fluid** Fluid under no shear stress is static and the normal to the surface and the stress vector are collinear. The gradient, divergence and Stokes theorems are the relations used to find the point or field equations of mass and momentum conservation from the integral equations. These equations are integrated for arbitrary control volumes to provide the density, pressure and velocity fields.	Sec. 2.2 and 3.3.	1. Use the divergence theorem to show that $\int_A \hat{r} x S \hat{n} dA = \int_V \hat{r} x \nabla S dV$ 2. Problem 2.1.
01.15	**6. Forces on Submerged Surfaces**. Euler's first equation is integrated to calculate the forces on curved surfaces and to derive Archemides Principle. The projected area theory simplifies the calculations for complex geometries.	Sec. 2.3-2.7	Problem 2.8
01.16	**7. Kinematics**. Material and spatial descriptions of moving particles are the foundation for determining their position, velocity and acceleration during deformation. When the identified particles in the observed system do not change, the derivative is defined as the material derivative. If different particles are considered over time, the derivative is defined as the general derivative. Streamlines, path lines and streak lines differ according to the particles observed and tracked.	Pages 75-84 and 97-98.	Problems 3.1 and 3.2
01.20	**8. Transport Theorems and Mass Conservation**. Time derivatives of material volume integrals have already appeared in Euler's first and second laws as well as the mass conservation equation. To develop the microscopic point equations of motion and mass conservation, the order of differentiation and differentiation must be reversed, and in so doing time derivatives of volume integrals can be formed as integrals of derivatives. Furthermore, in macroscopic analysis presented later in the course, derivatives of volume integrals which are not material, will appear and Leibnitz rule will be used reverse the operations.	Secs. 3.4 and 3.5	Show the development the Special form of the Reynold's Transport Theorem (Hint: let S=?**v**).
01.21	**9. Application of Macroscopic Mass Balance.**	Sec. 7.1	Flow in veins and arteries is a transient process in which elastic conduits expand and contract. Consider an artery having a radial velocity at the inner radius of 0.012 cm/s. The length L of the artery is 13 cm and the volumetric flow rate at the entrance of the artery is 0.3 cm^3/s. At some instant of time, the inner radius of the artery is 0.15 cm. At that particular moment, what is the volumetric flow rate at the artery exit? (hint: dA=rd?L in which A is cross sectional area, r is the radius and ? is the angle in radians.)

01.22	**10. Stream Function**. The velocity field is calculated as the gradient of a scalar field for the special case of steady incompressible flow. The resulting scalar stream function satisfies the continuity equation exactly.	Pages 100-104.	If a stream function exists for a velocity field of $v_x = a(x^2 - y^2)$ $v_y = -2axy$ $v_z = 0$ in which a is parameter, find the stream function ?.
01.23 **No Class**	**Monday classes meet on this Friday.**		
01.27	**11. Cauchy's First and Second Equations**. The special form of the Reynolds transport theorem applied to Euler's first law transforms the integral equation into the point or field equation of linear momentum including the stress tensor. Representing the stress vector in Euler's first law as a function of the stress tensor allowed the transformation of the area integral into a volume integral. The divergence theorem, Reynolds transport theorem, Cauchy's lemma and Cauchy's first equation transform Euler's second law, an integral equation, into the point or field equation showing the symmetry of the stress tensor. This is Cauchy's second equation.		Use the special form of the Reynolds transport theorem to show that $\frac{d}{dt}\int_{> m(t)} \hat{r}x\rho\hat{v}dV = \int_{> m(t)} \rho\frac{D}{Dt}(\hat{r}x\hat{v})dV$
01.28	**12. Viscous Stress, Rate of Strain and Vorticity Tensors**. The viscous stress tensor is defined as a function of pressure and the rate of deformation of the fluid via the strain tensor. The velocity gradient tensor is decomposed into the symmetric rate of strain tensor and the vorticity tensor which does not contribute to the rate of deformation of the fluid.	Pages 128-134.	
01.29	**13. Physical Interpretation of the Rate of Strain and the Vorticity Tensors**. From the velocity gradient calculate the rate of stretching of a line element, the rate of angle change between material line elements and the rate of rotation (rigid body rotation). Relate the vorticity tensor and vorticity vector.	Sec. 5.5	For the plane Couette Flow illustrated in Figure 5.3-1, find the rate of strain in the ? direction, ? that maximizes rate of strain, the rate of decrease of the angle between unit vectors initially in the x and y directions and the components of the vorticity vector.
01.30	**14. Velocity Potential and Stream Function**. Stokes theorem is used to show that irrotationality and simple topological connectivity are the conditions for the velocity to be determined as the gradient of a three dimensional scalar field f. The differential of the scalar field f is exact and "conservative." Because the stream function ? and f are harmonic and they satisfy the Cauchy-Riemann equations, the lines of constant ? and f are orthogonal.		If a velocity potential function exists for a velocity field of $v_x = a(x^2 - y^2)$ $v_y = -2axy$ $v_z = 0$ in which a is parameter, find the velocity potential function f.

02.03	**15. Newton's Law of Viscosity and the Equations of Motion**. The linear tensor equation of viscosity is limited to isotropic fluids. Substituting this form into the stress equations of motions provides the Navier-Stokes Equation.	Pages 14-16, 139-146. Sec. 5.4	
02.04	**MIDTERM (Through Lecture 15)**		
02.05	**16. Application**. Uniformly accelerated flow.	Pages 166-169.	Problem 5-12.
02.06	**17. Application**. One-dimensional laminar flow.	Pages 169-173.	Problem 5-14.
02.10	**18. Application**. Transient flow and the suddenly accelerated flat plat. The von Karman-Pohlhausen integral method provides an estimate of the propagation of a disturbance at a boundary as well as the velocity profile.	Sec. 11.2	Problem 11-1.
02.11	**19. Application**. Laminar boundary layer equations. Several restrictions are made to the conservation and state equations which allow for an approximate solution.	Class notes	
02.12	**20. Application**. The von Karman-Pohlhausen integral method provides an estimate of the velocity profile for laminar boundary layer flow.	Class notes	Starting with the integral motion equation $\frac{d}{dx}\int_{y=0}^{y=\delta_H} v_x dx - u_\infty \int_{y=0}^{y=\delta_H} v_x dx = \nu \left.\frac{\partial v_x}{\partial y}\right\|_{y=0}$, represent the velocity component as a third order polynomial in $\frac{y}{\delta_H}$ and show that $\delta_H = 4.64\sqrt{\frac{v_x}{u_\infty}}$.
02.13	**21. Dimensional Analysis**. The governing equations are made dimensionless to reduce the number of experiments needed to solve flow problems that are not susceptible to analysis.	Sec. 5.5	Work the racing sloop example problem which starts on page 163. Show all the steps, do not just copy what is in the text. Discuss geometric and dynamic similarity.
02.17	**22. Transition and Turbulent Flow.** Small disturbances in the laminar flow region create velocity variations in time. In the laminar boundary layer fluid parcels follow a straight path, deform and rotate while in the transition region the path is curvilinear and parcels oscillate. In the turbulent region, the path is undefined and the parcel rotates unpredictably. Velocity is decomposed into time average and turbulent fluctuation terms. Because the time scales for each term are disparate, the time average of the time average velocity is equal to the time average velocity.	Sec. 6.1 and Class notes	Problem 6.1
02.18	**23. Time Averaged Continuity and Navier Stokes Equations**. Leibnitz rule is used to begin deriving the time averaged equations of incompressible flow.	Sec. 6.2	Problem 6.2

02.19	**24. Time Averaged Continuity and Navier Stokes Equations**. The time-averaged equations of motion indicates that turbulent flow can be treated in the same way as laminar flows provided the pressure and velocity are replaced by the time-averaged quantities and the viscous stress tensor is replaced by the total time-averaged stress tensor which is the sum of the viscous and turbulent stress tensors.	Sec. 6.2	
02.20	**25. Physical Interpretation of Turbulence**. Turbulence is generated near the tube wall and the intensity falls off toward the center of the tube. In the central region the generating force (shear deformation) decreases and viscous forces tend to reduce turbulence.	Sec. 6.3	
02.24	**26. Eddy Viscosity and Prandtl's Mixing Length Theory**. As an analog to laminar flow, a turbulent or Eddy viscosity was developed by Prandtl through a simplified interpretation of turbulent momentum transfer.	Sec. 6.4	
02.25	**27. Application to Turbulent Pipe Flow Velocity Profiles**. Mixing length theory is applied to turbulent pipe flow to calculate the time averaged velocity profile.	Sec. 6.5	
02.26	**28. Macroscopic Momentum Balance**. To solve more complex problems that are not subject to microscopic analysis, we supplement information from intuition and experimentation and find solutions which are correct on the average. The governing equations are satisfied for a control volume rather than point-wise. Information lost through integration must be replaced by intuition, experiment or analysis at smaller length scales.	Secs. 7.1 and 7.2	Derive Euler's First Law $\frac{d}{dt}\int_{\mathcal{V}_m(t)} \rho\hat{v}dV = \int_{\mathcal{V}_m(t)} \rho\hat{b}dV + \int_{\mathcal{A}_m(t)} \hat{t}_{(\hat{n})}dA$ starting from the following axiomatic statement of the linear momentum principle $\frac{d}{dt}\int_{\mathcal{V}_a(t)} \rho\hat{v}dV + \int_{\mathcal{A}_m(t)} \rho\hat{v}(\hat{v}-\hat{w})\bullet\hat{n}dA =$ $\int_{\mathcal{V}_a(t)} \rho\hat{b}dV + \int_{\mathcal{A}_a(t)} \hat{t}_{(\hat{n})}dA$
02.27	**29. Application**. Jets and Plates. Force exerted by the fluid on a plate is calculated using the macroscopic momentum conservation principle. Assumptions that must be made in order to arrive at the simple solution will be identified.	Secs. 7.1 and 7.8	Problem 7-6. Ignore the comments about the energy equation and use the momentum balance to solve this problem. Microscopic scale information lost by integration over area is recovered or justified by the statement that "viscous surface forces can be neglected."
03.02	**30. Bernoulli's Equation.** Bernoulli's is obtained by first extracting the component of the Navier-Stokes equation tangent to a streamline, simplifying that result by neglecting the local acceleration and viscous effects, and then integrating along a streamline.	Pages 230-235.	Problems 7-5, 7-7 (Use Torricelli's equation with $C_d = 1$ so that this problem can be solved using a macroscopic momentum balance analysis), and 7-19. Torricelli's equation: $Q = A_o C_d \sqrt{2gh(t)}$

03.03	**31. Moving Control Volumes and Inertial Frames**. Because the mass and the linear and angular momentum balance equations are valid in inertial frames, careful selection of the inertial frame can simplify problem solving. The problem of finding the force exerted by a plane jet impinging on a curved vane is illustrative.	Sec. 7.9 but ignore the energy balance discussion.	Problems 7-4, 7-16 and 7-17. Note that problem 7-17 should read: Does a converging nozzle on a garden hose place the hose (at the junction between the hose and the nozzle) in tension or compression?
03.04	**32. Mechanical Energy Balance**. By forming the scalar product with the velocity vector the necessity of evaluating terms in the macroscopic momentum balance equation at solid surfaces is eliminated but a viscous dissipation term arises which must be evaluated experimentally. Keep in mind that the momentum and mechanical energy balances come from the same physical principle but the assumptions in making approximate solutions are different.	Sec. 7.3	Problems 7-3 and 7-20
03.05	**33. Application**. Sudden expansion in a pipeline.	Sec. 7.5 and pages 311-316.	Problems 7-9 and 7-10.
03.09	**34. Turbulent flow in pipes**. We begin developing a consistent method of interpreting experimental data.	Pages 285-293.	
03.10	**MIDTERM** (Through lecture 33)		
03.11	**35. Friction Factors**. Experimental data is interpreted to generalize application of the momentum balance equation to non-circular conduits as well as flows around spheres and cylinders.	Sec. 8.2	Problems 8-3 and 8-4.
03.12	**36. Pipeline Design.** The macroscopic mechanical energy balance equation simplifies calculation of headloss in a pipeline. The energy and momentum balance equations are combined to arrive at the simplified approach.	Sec. 8.3	Problem 8-9.
03.16	Review		
03.24	**FINAL EXAM** (Comprehensive) 4-6 pm, Chem 166		